Probability Tutorials: Notations

1. Tutorial 1

$\stackrel{\triangle}{=}$: equality which is true by definition, hence always true.

Ω : an arbitrary set.

$\mathcal{P}(\Omega)$: the power set of Ω, i.e. the set of all subsets of Ω.

$\mathcal{D}$: a set of subsets of Ω, also a Dynkin system on Ω.

$\mathcal{F}$: a set of subsets of Ω, also a σ-algebra on Ω.

$\Omega \in \mathcal{D}$: Ω is an element of the set $\mathcal{D}$.

A, B : arbitrary subsets of Ω.

$(A_n)_{n\geq 1}$: a sequence of subsets of Ω.

$A \subseteq B$: A is a subset of B, i.e. $x \in A \Rightarrow x \in B$.

$B \setminus A$: set difference defined by $B \setminus A = \{x \in B : x \notin A\}$.

$\cup_{n=1}^{+\infty} A_n$: union of all A_n's, $\cup_{n=1}^{+\infty} A_n = \{x : \exists n \geq 1, x \in A_n\}$.

A^c : the complement of A in Ω, $A^c = \{x \in \Omega : x \notin A\}$.

$A \cup B$: union of A and B, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

$A \cap B$: intersection of A and B, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
$(\mathcal{D}_i)_{i \in I}$: a family of Dynkin systems on Ω, indexed by a set I.
$\cap_{i \in I} \mathcal{D}_i$: intersection of all $\mathcal{D}_i$'s, $\cap_{i \in I} \mathcal{D}_i = \{A : \forall i \in I, A \in \mathcal{D}_i\}$.
$(\mathcal{F}_i)_{i \in I}$: a family of σ-algebras on Ω, indexed by a set I.
$\cap_{i \in I} \mathcal{F}_i$: intersection of all $\mathcal{F}_i$'s, $\cap_{i \in I} \mathcal{F}_i = \{A : \forall i \in I, A \in \mathcal{F}_i\}$.
$\mathcal{A}$: a set of subsets of Ω, a subset of $\mathcal{P}(\Omega)$.
$D(\mathcal{A})$: the set of all Dynkin systems on Ω, containing $\mathcal{A}$.
$\mathcal{D}(\mathcal{A})$: the Dynkin system on Ω, generated by $\mathcal{A}$.
$\sigma(\mathcal{A})$: the σ-algebra on Ω, generated by $\mathcal{A}$.
$\mathcal{C}$: a set of subsets of Ω, also a π-system on Ω.

2. Tutorial 2

Ω : an arbitrary set.
$\mathcal{P}(\Omega)$: the power set of Ω, i.e. the set of all subsets of Ω.
$\emptyset$: the empty set, i.e. the only set with no elements.
$B \setminus A$: set difference defined by $B \setminus A = \{x \in B : x \notin A\}$.
$\uplus$: union of pairwise disjoint sets.

$\mathcal{R}$: a set of subsets of Ω, also a ring on Ω.
$(\mathcal{R}_i)_{i\in I}$: a family of rings on Ω, indexed by a set I.
$\mathcal{A}$: a set of subsets of Ω, a subset of $\mathcal{P}(\Omega)$.
$R(\mathcal{A})$: the set of all rings on Ω, containing $\mathcal{A}$.
$\mathcal{R}(\mathcal{A})$: the ring on Ω, generated by $\mathcal{A}$.
μ : a measure defined on a set of subsets of Ω.
$[0, +\infty]$: the set $\mathbf{R}^+ \cup \{+\infty\}$.
$\mathcal{R}(\mathcal{S})$: the ring on Ω, generated by the semi-ring $\mathcal{S}$.
$\bar{\mu}, \bar{\mu}'$: measures defined on the ring $\mathcal{R}(\mathcal{S})$.
$\bar{\mu}_{|\mathcal{S}}, \bar{\mu}'_{|\mathcal{S}}$: the restrictions of $\bar{\mu}$ and $\bar{\mu}'$ to the smaller domain $\mathcal{S}$.
μ^* : an outer-measure on Ω.
$\Sigma(\mu^*), \Sigma$: the σ-algebra on Ω, associated with μ^*.
A, B, T : arbitrary subsets of Ω.
A^c : the complement of A in Ω, $A^c = \{x \in \Omega : x \notin A\}$.
$\mu^*_{|\Sigma}$: the restriction of μ^* to the smaller domain Σ.
$\sigma(\mathcal{R})$, $\sigma(\mathcal{R}(\mathcal{S}))$, $\sigma(\mathcal{S})$: σ-algebras on Ω, generated by $\mathcal{R}$, $\mathcal{R}(\mathcal{S})$, $\mathcal{S}$.
μ' : a measure defined on $\sigma(\mathcal{R})$, or $\sigma(\mathcal{S})$.
$\mu'_{|\mathcal{R}}$, $\mu'_{|\mathcal{S}}$: the restrictions of μ' to the smaller domains $\mathcal{R}$ and $\mathcal{S}$.

3. Tutorial 3

Ω : an arbitrary set.
$\mathcal{P}(\Omega)$: the power set of Ω, i.e. the set of all subsets of Ω.
$\mathcal{A}$: a set of subsets of Ω.
μ : a finitely additive map on $\mathcal{A}$ or a measure on $\mathcal{F}$.
$\uplus$: a union of pairwise disjoint sets.
A, A_i, A_n : arbitrary substets of Ω.
$a \vee b$: the largest of a and b, $a \vee b = \max(a, b)$.
$a \wedge b$: the smallest of a and b, $a \wedge b = \min(a, b)$.
$\mathcal{S}$: the semi-ring $\mathcal{S} = \{]a, b]\,,\, a, b \in \mathbf{R}\}$, or a semi-ring on Ω.
$\mathcal{R}(\mathcal{S})$: the ring generated by $\mathcal{S}$.
$\bar{\mu}$: a finitely additive map defined on $\mathcal{R}(\mathcal{S})$.
F: a right-continuous and non-decreasing map defined on $\mathbf{R}$ or $\mathbf{R}^+$.
$\mathcal{T}$: a topology on Ω.
$(\Omega, \mathcal{T})$: a topological space.
$\mathcal{B}(\Omega)$: the Borel σ-algebra on $(\Omega, \mathcal{T})$.
$\mathbf{R}$: the real line $\mathbf{R} =]-\infty, +\infty[$.

$\mathbf{R}^+$: the subset of $\mathbf{R}$, $\mathbf{R}^+ = [0, +\infty[$.
$\mathcal{T}_{\mathbf{R}}$: the usual topology on $\mathbf{R}$.
$\mathcal{B}(\mathbf{R})$: the Borel σ-algebra on $\mathbf{R}$.
$\mathcal{B}(\mathbf{R}^+)$: the Borel σ-algebra on $\mathbf{R}^+$.
$\mathbf{Q}$: the set of all rational numbers.
$\sigma(\mathcal{S})$: the σ-algebra generated by $\mathcal{S}$.
$\mathcal{F}$: a σ-algebra on Ω.
$(\Omega, \mathcal{F})$: a measurable space.
$(\Omega, \mathcal{F}, \mu)$: a measure space.
$A_n \uparrow A$: for all $n \geq 1$, $A_n \subseteq A_{n+1}$ and $A = \cup_{n=1}^{+\infty} A_n$.
$A_n \downarrow A$: for all $n \geq 1$, $A_{n+1} \subseteq A_n$ and $A = \cap_{n=1}^{+\infty} A_n$.
$\mathcal{D}_n$: a Dynkin system on $\mathbf{R}$ or $\mathbf{R}^+$.
μ_1, μ_2 : measures defined on $\mathcal{B}(\mathbf{R})$ or $\mathcal{B}(\mathbf{R}^+)$.
dF : the Stieltjes measure on $\mathcal{B}(\mathbf{R})$ or $\mathcal{B}(\mathbf{R}^+)$ associated with F.
dx : the Lebesgue measure on $\mathcal{B}(\mathbf{R})$.
$F(x_0-)$: the left limit of F at $x = x_0$.
Ω' : a subset of Ω.
$\mathcal{A}_{|\Omega'}$: the trace of $\mathcal{A}$ on Ω', $\mathcal{A}_{|\Omega'} = \{A \cap \Omega' : A \in \mathcal{A}\}$.

$\mathcal{T}_{|\Omega'}$: the toplogy on Ω', induced by the topology $\mathcal{T}$ on Ω.
$\sigma(\mathcal{A})$: the σ-algebra on Ω generated by $\mathcal{A}$.
$\sigma(\mathcal{A}_{|\Omega'})$: the σ-algebra on Ω' generated by $\mathcal{A}_{|\Omega'}$.
$\sigma(\mathcal{A})_{|\Omega'}$: the trace of $\sigma(\mathcal{A})$ on Ω'.
$\mathcal{B}(\Omega)_{|\Omega'}$: the trace of $\mathcal{B}(\Omega)$ on Ω'.
$\mathcal{B}(\Omega')$: the Borel σ-algebra on $(\Omega', \mathcal{T}_{|\Omega'})$.
$\mathcal{F}_{|\Omega'}$: the trace of $\mathcal{F}$ on Ω'.
$\mu_{|\Omega'}$: the restriction of μ to $\mathcal{F}_{|\Omega'}$, when $\Omega' \in \mathcal{F}$.

4. Tutorial 4

$f : A \to B$: a map defined on A with values in B.
$f(A')$: direct image of A' by f, $f(A') = \{f(x) : x \in A'\}$.
$f^{-1}(B')$: inverse image of B' by f, $f^{-1}(B') = \{x \in A : f(x) \in B'\}$.
$\{f \in B'\}$: same as $f^{-1}(B')$.
$(\Omega, \mathcal{T})$, $(S, \mathcal{T}_S)$: topological spaces.
(E, d), (F, δ) : metric spaces.
$B(x, \epsilon)$: the open ball on E, $B(x, \epsilon) = \{y \in E : d(x, y) < \epsilon\}$.

$\mathcal{T}_E^d$: the metric topology on E, associated with the metric d.
$d_{|F}$: restriction of the metric d to $F \times F$, when $F \subseteq E$.
$\mathcal{T}_F$, $(\mathcal{T}_E^d)_{|F}$: the topology on F, induced by the metric topology $\mathcal{T}_E^d$.
$\mathcal{T}'_F$, $\mathcal{T}_F^{d_{|F}}$: the metric topology on F, associated with the metric $d_{|F}$.
$\bar{\mathbf{R}}$: the extended real line, $\bar{\mathbf{R}} = \mathbf{R} \cup \{-\infty, +\infty\} = [-\infty. + \infty]$.
$\mathcal{T}_{\bar{\mathbf{R}}}$: the usual topology on $\bar{\mathbf{R}}$.
$\mathcal{T}_{\mathbf{R}}$: the usual topology on $\mathbf{R}$.
$(\mathcal{T}_{\bar{\mathbf{R}}})_{|\mathbf{R}}$: the topology on $\mathbf{R}$, induced by the usual topology on $\bar{\mathbf{R}}$.
$\mathcal{B}(\mathbf{R})$: the Borel σ-algebra on $\mathbf{R}$.
$\mathcal{B}(\bar{\mathbf{R}})$: the Borel σ-algebra on $\bar{\mathbf{R}}$.
$\mathcal{B}(\bar{\mathbf{R}})_{|\mathbf{R}}$: the trace of $\mathcal{B}(\bar{\mathbf{R}})$ on $\mathbf{R}$.
$\mathcal{T}_{\bar{\mathbf{R}}}^d$: the metric topology on $\bar{\mathbf{R}}$ associated with the metric d.
$(\Omega, \mathcal{F})$, (S, Σ), (S_1, Σ_1) : measurable spaces.
Σ', $\Sigma_{|S'}$: the trace of Σ on S'.
$g \circ f$: the composition of g and f, defined by $g \circ f(x) = g(f(x))$.
$\mathcal{A}$: a set of subsets of S.
$\sigma(\mathcal{A})$: the σ-algebra on S generated by $\mathcal{A}$.

$\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4$: set of subsets of $\bar{\mathbf{R}}$.
$\{f \le c\}$: the inverse image of $[-\infty, c]$ by f.
$\{f < c\}$: the inverse image of $[-\infty, c[$ by f.
$\{c \le f\}$: the inverse image of $[c, +\infty]$ by f.
$\{c < f\}$: the inverse image of $]c, +\infty]$ by f.
$\inf_{n\ge 1} v_n$: the greatest lower-bound of $\{v_n : n \ge 1\}$.
$\sup_{n\ge 1} v_n$: the smallest upper-bound of $\{v_n : n \ge 1\}$.
$\liminf v_n$: the lower limit of $(v_n)_{n\ge 1}$ as $n \to +\infty$.
$\limsup v_n$: the upper limit of $(v_n)_{n\ge 1}$ as $n \to +\infty$.
$\lim v_n$: the limit of $(v_n)_{n\ge 1}$ as $n \to +\infty$.
f^+ : the positive part of f, $f^+ = \max(f, 0)$.
f^- : the negative part of f, $f^- = \max(-f, 0)$.
$\bar{A}$: the closure of A in $(\Omega, \mathcal{T})$.
$d(x, A)$: the distance from x to A, $d(x, A) = \inf\{d(x, y) : y \in A\}$.
$\lim f_n$: simple limit of $(f_n)_{n\ge 1}$, defined by $(\lim f_n)(\omega) = \lim f_n(\omega)$.
$\mathbf{C}$: the set of complex numbers.
$Re(f)$: the real part of f.
$Im(f)$: the imaginary part of f.

5. Tutorial 5

$(\Omega, \mathcal{F}, \mu)$: an arbitrary measure space.
1_A : the characteristic function of $A \subseteq \Omega$.
$\uplus$: a union of pairwise disjoint sets.
$I^\mu(s)$: the integral w.r. to μ of the simple function s on $(\Omega, \mathcal{F})$.
$\int f d\mu$: the Lebesgue integral of f with respect to μ.
$v_n \uparrow v$: for all $n \geq 1$, $v_n \leq v_{n+1}$ and $v = \sup_{n\geq 1} v_n$.
$f_n \uparrow f$: for all $\omega \in \Omega$, $f_n(\omega) \uparrow f(\omega)$.
$A_n \uparrow A$: for all $n \geq 1$, $A_n \subseteq A_{n+1}$ and $A = \cup_{n=1}^{+\infty} A_n$.
$\mathcal{P}(\omega)$, μ-a.s. : the property $\mathcal{P}$ holds μ-almost surely.
$\mathcal{F}_{|A}$: the trace of $\mathcal{F}$ on $A \subseteq \Omega$.
$\mu_{|A}$: the restriction of μ to $\mathcal{F}_{|A}$, when $A \in \mathcal{F}$.
$f_{|A}$: the restriction of f to A.
μ^A : the measure defined on $\mathcal{F}$ by $\mu^A(E) = \mu(A \cap E)$.
$\int_A f d\mu$: the partial Lebesgue integral of f over A with respect to μ.
$L^1_{\mathbf{R}}(\Omega, \mathcal{F}, \mu)$: set of **R**-valued, measurable maps with $\int |f| d\mu < +\infty$.
$L^1_{\mathbf{C}}(\Omega, \mathcal{F}, \mu)$: set of **C**-valued, measurable maps with $\int |f| d\mu < +\infty$.

6. Tutorial 6

I : an arbitrary non-empty set.
$(\Omega_i)_{i\in I}$: a familiy of sets indexed by I.
$\prod_{i\in I}\Omega_i$: the cartesian product of the family $(\Omega_i)_{i\in I}$.
Ω^I : the cartesian product when $\Omega_i = \Omega$, for all $i \in I$.
$\prod_{n=1}^{+\infty}\Omega_n$: the cartesian product when $I = \mathbf{N}^*$.
$\Omega_1 \times \ldots \times \Omega_n$: the cartesian product when $I = \mathbf{N}_n$.
$\mathbf{N}$: the set $\mathbf{N} = \{0, 1, 2, \ldots\}$.
$\mathbf{N}^*$: the set $\mathbf{N}^* = \{1, 2, 3, \ldots\}$.
$\mathbf{N}_n$: the set $\mathbf{N}_n = \{1, 2, \ldots, n\}$.
$(I_\lambda)_{\lambda\in\Lambda}$: a partition of the set I.
$(\mathcal{E}_i)_{i\in I}$: a family, where each $\mathcal{E}_i$ is a set of subsets of Ω_i.
$\prod_{i\in I} A_i$: a rectangle of the family $(\mathcal{E}_i)_{i\in I}$.
$\coprod_{i\in I}\mathcal{E}_i$: the set of all rectangles of the family $(\mathcal{E}_i)_{i\in I}$.
$\mathcal{E}_1 \amalg \ldots \amalg \mathcal{E}_n$: the set of all rectangles when $I = \mathbf{N}_n$.
$(\Omega_i, \mathcal{F}_i)_{i\in I}$: a family of measurable spaces indexed by I.
$\coprod_{i\in I}\mathcal{F}_i$: the set of measurable rectangles, the rectangles of $(\mathcal{F}_i)_{i\in I}$.

$\otimes_{i\in I}\mathcal{F}_i$: the product σ-algebra of $(\mathcal{F}_i)_{i\in I}$ on $\Pi_{i\in I}\Omega_i$.
$\sigma(\coprod_{i\in I}\mathcal{F}_i)$: the σ-algebra generated by the measurable rectangles.
$\mathcal{F}_1\otimes\ldots\otimes\mathcal{F}_n$: the product σ-algebra when $I=\mathbf{N}_n$.
$\sigma(\mathcal{E}_i)$: the σ-algebra on Ω_i, generated by $\mathcal{E}_i$.
$\otimes_{i\in I}\sigma(\mathcal{E}_i)$: the product σ-algebra of $(\sigma(\mathcal{E}_i))_{i\in I}$ on $\Pi_{i\in I}\Omega_i$.
$\coprod_{i\in I}\sigma(\mathcal{E}_i)$: the set of measurable rectangles of $(\sigma(\mathcal{E}_i))_{i\in I}$.
$\mathcal{T}_{\mathbf{R}}$: the usual toplogy on $\mathbf{R}$.
$\mathcal{T}_{\mathbf{R}}\amalg\ldots\amalg\mathcal{T}_{\mathbf{R}}$: set of rectangles when $I=\mathbf{N}_n$ and $\mathcal{E}_i=\mathcal{T}_{\mathbf{R}}$.
$\mathcal{A}$: a set of subsets of Ω.
$\mathcal{T}(\mathcal{A})$: the topology on Ω, generated by $\mathcal{A}$.
$(\Omega_i,\mathcal{T}_i)_{i\in I}$: a family of topological spaces indexed by I.
$\coprod_{i\in I}\mathcal{T}_i$: the set of rectangles of $(\mathcal{T}_i)_{i\in I}$.
$\odot_{i\in I}\mathcal{T}_i$: the product topology of $(\mathcal{T}_i)_{i\in I}$ on $\Pi_{i\in I}\Omega_i$.
$\mathcal{B}(\Omega_i)$: the Borel σ-algebra on $(\Omega_i,\mathcal{T}_i)$.
$\otimes_{i\in I}\mathcal{B}(\Omega_i)$: product σ-algebra of $(\mathcal{B}(\Omega_i))_{i\in I}$ on $\Pi_{i\in I}\Omega_i$.
$\mathcal{H}$: a countable base of $(\Omega,\mathcal{T})$.
$\mathcal{B}(\Pi_{i\in I}\Omega_i)$: the Borel σ-algebra for the product topology.

7. Tutorial 7

E^{ω_1} : ω_1-section of a subset E of $\Omega_1 \times \Omega_2$.
$\mathcal{F}_1 \amalg \mathcal{F}_2$: set of measurable rectangles of $\mathcal{F}_1$ and $\mathcal{F}_2$.
$\mathcal{F}_1 \otimes \mathcal{F}_2$: product σ-algebra of $\mathcal{F}_1$ and $\mathcal{F}_2$.
$\mathcal{B}(E)$: Borel σ-algebra on a metric space (E, d).
$\Omega_n \uparrow \Omega$: for all $n \geq 1$, $\Omega_n \subseteq \Omega_{n+1}$ and $\Omega = \cup_{n=1}^{+\infty} \Omega_n$.
$\mu_1 \otimes \ldots \otimes \mu_n$: product of σ-finite measures.
dx^n : the Lebesgue measure on $(\mathbf{R}^n, \mathcal{B}(\mathbf{R}^n))$.
$\mathbf{N}_n$: the set $\{1, \ldots, n\}$.
σ : a permutation, i.e. a bijection $\sigma : \mathbf{N}_n \to \mathbf{N}_n$.
$f_p \uparrow f$: For all $p \geq 1$, $f_p \leq f_{p+1}$ and $f = \lim f_p$.
$\int_{\Omega_2} f(\omega, x) d\mu_2(x)$: the integral of $f(\omega, \bullet)$ w.r. to μ_2, $\omega \in \Omega_1$.

8. Tutorial 8

$\lim_{x \downarrow\downarrow x_0} \phi(x)$: the limit of $\phi(x)$ as $x \to x_0$ with $x_0 < x$.
$\mathcal{T}_{|K}$: the induced topology on K.

$\delta(A)$: the diameter of a set A.
$\inf_{x\in\Omega} f(x)$: the infimum of $f(\Omega)$.
$\sup_{x\in\Omega} f(x)$: the supremum of $f(\Omega)$.
$f'(c)$: the derivative of f evaluated at c.
$f^{(k)}(a)$: the k^{th} derivative of f evaluated at a.
C^n: [of class] for all $k \leq n$, $f^{(k)}$ exists and is continuous.
$(\Omega, \mathcal{F}, P)$: a probability space.
(S, Σ) : a measurable space.
$E[X]$: the expectation of the random variable X.
$\phi \circ X$: the composition $\phi \circ X(\omega) = \phi(X(\omega))$.

9. Tutorial 9

$(\Omega, \mathcal{F}, \mu)$: a measure space.
$L^p_{\mathbf{R}}(\Omega, \mathcal{F}, \mu)$: set of **R**-valued measurable maps f, with $\|f\|_p < +\infty$.
$L^p_{\mathbf{C}}(\Omega, \mathcal{F}, \mu)$: set of **C**-valued measurable maps f, with $\|f\|_p < +\infty$.
$\|f\|_p$: p-norm of f. For $p \in [1, +\infty[$, $\|f\|_p = (\int |f|^p d\mu)^{1/p}$.
$\|f\|_\infty$: ∞-norm of f. $\|f\|_\infty = \inf\{M \in \mathbf{R}^+ : |f| \leq M, \mu\text{-a.s.}\}$.

$B(f, \epsilon)$: the open ball in $L^p_{\mathbf{R}}(\Omega, \mathcal{F}, \mu)$ or $L^p_{\mathbf{C}}(\Omega, \mathcal{F}, \mu)$.
$x_n \stackrel{\mathcal{T}}{\to} x$: $(x_n)_{n\geq 1}$ converges to x, with respect to the topology $\mathcal{T}$.
$f_n \stackrel{L^p}{\to} f$: $(f_n)_{n\geq 1}$ converges to f in L^p. $\|f_n - f\|_p \to 0$.
$f_n \to f$: $(f_n)_{n\geq 1}$ converges to f, simply: $f_n(x) \to f(x)$ for all x.
$f_n \to f$, μ-a.s. : $f_n(x) \to f(x)$ for μ-almost all x.
$(f_{n_k})_{k\geq 1}$: a sub-sequence of $(f_n)_{n\geq 1}$.

10. Tutorial 10

$\mathbf{K}$: the field $\mathbf{R}$ or $\mathbf{C}$.
$\mathbf{N}^*$: the set of positive integers, $\mathbf{N}^* = \{1, 2, 3, \ldots\}$.
$\mathcal{T}_{\mathbf{R}^n}$: usual topology on $\mathbf{R}^n$.
$\mathcal{T}_{\bar{\mathbf{R}}}$: usual topology on $\bar{\mathbf{R}}$.
$x_n \stackrel{\mathcal{T}}{\to} x$: convergence with respect to a topology $\mathcal{T}$.
$d_{\mathbf{C}^n}$: usual metric on $\mathbf{C}^n$.
$d_{\mathbf{R}^n}$: usual metric on $\mathbf{R}^n$.
$\delta(A)$: diameter of A, $\delta(A) = \sup\{d(x, y) : x, y \in A\}$.

$\bar{F}$: closure of the set F.
$\bar{z}$: complex conjugate of z. If $z = a + ib$, $\bar{z} = a - ib$.
$\langle \cdot, \cdot \rangle$: an inner-product on a **K**-vector space.
$\| \cdot \|$: the norm induced by an inner product, $\| \cdot \| = \sqrt{\langle \cdot, \cdot \rangle}$.
$\mathcal{T}_{\langle \cdot, \cdot \rangle}$: norm topology induced by an inner-product.
$\mathcal{G}^{\perp}$: orthogonal of a set $\mathcal{G}$ w.r. to some inner-product.
$[f]$: μ-almost sure equivalence class of f in $L^2_{\mathbf{K}}(\Omega, \mathcal{F}, \mu)$.

11. Tutorial 11

$\mathbf{N}^*$: the set of positive integers $\mathbf{N}^* = \{1, 2, 3, \ldots\}$.
$\mathbf{Z}$: the set of integers $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
$(\Omega, \mathcal{F})$: a measurable space.
σ : a bijection between $\mathbf{N}^*$ and itself.
$\uplus_{n \geq 1}$: a countable union of pairwise disjoint sets.
dx : the Lebesgue measure on $(\mathbf{R}^n, \mathcal{B}(\mathbf{R}^n))$.
$M^1(\Omega, \mathcal{F})$: set of complex measures on $(\Omega, \mathcal{F})$.

$|z|$: modulus of complex number z.
$|\mu(E)|$: modulus of complex number $\mu(E)$.
$|\mu|$: total variation of complex measure μ.
$|\mu|(E)$: $|\mu|$-measure of the measurable set E.
μ^+ : positive part of signed measure μ, $\mu^+ = (|\mu| + \mu)/2$.
μ^- : negative part of signed measure μ, $\mu^- = (|\mu| - \mu)/2$.

12. Tutorial 12

$(\Omega, \mathcal{F})$: a measurable space.
$\nu << \mu$: the measure ν is absolutely continuous w.r. to μ.
$\limsup_{n\geq 1} E_n$: the set $\cap_{n\geq 1} \cup_{k\geq n} E_k$, also denoted $\{E_n : \text{i.o.}\}$.
$M^1(\Omega, \mathcal{F})$: set of complex measures on $(\Omega, \mathcal{F})$.
$|\nu|$: total variation of complex measure ν.
$E_n \uparrow E$: $E_n \subseteq E_{n+1}$ for all $n \geq 1$, and $E = \cup_{n\geq 1} E_n$.
u^+ : positive part of function u, $u^+ = u \vee 0 = \max(u, 0)$.
μ^+ : positive part of signed measure μ, $\mu^+ = (|\mu| + \mu)/2$.
$\mathcal{F}_{|A}$: trace of σ-algebra $\mathcal{F}$ on A, $\mathcal{F}_{|A} = \{A \cap E : E \in \mathcal{F}\}$.

$\mu_{|A}$: restriction of μ to $\mathcal{F}_{|A}$.
μ^A : the complex measure $\mu(A\cap\cdot)$ on $(\Omega,\mathcal{F})$.
$|\mu^A|$: total variation of the complex measure μ^A on $(\Omega,\mathcal{F})$.
$|\mu_{|A}|$: total variation of the complex measure $\mu_{|A}$ on $(A,\mathcal{F}_{|A})$.
$|\mu|^A$: the measure $|\mu|(A\cap\cdot)$.
$|\mu|_{|A}$: restriction of $|\mu|$ to $\mathcal{F}_{|A}$.
$f_{|A}$: restriction of the map f to A.
$\int f_{|A}d\mu_{|A}$: integral of $f_{|A}$ on the measure space $(A,\mathcal{F}_{|A},\mu_{|A})$.
$\mathcal{F}_1\otimes\ldots\otimes\mathcal{F}_n$: product of the σ-algebras $\mathcal{F}_1,\ldots,\mathcal{F}_n$.
$\|\mu\|$: total mass of total variation of μ, $\|\mu\|=|\mu|(\Omega)$.

13. Tutorial 13

$\mathbf{K}$: the field $\mathbf{R}$ or $\mathbf{C}$.
$S_{\mathbf{K}}(\Omega,\mathcal{F})$: set of $\mathbf{K}$-valued complex simple functions on $(\Omega,\mathcal{F})$.
$C^b_{\mathbf{K}}(\Omega)$: set of $\mathbf{K}$-valued continuous and bounded maps on Ω.
$M^1(\Omega,\mathcal{B}(\Omega))$: set of complex Borel measures on Ω.
$d(x,A)$: distance from x to A, $d(x,A)=\inf\{d(x,y):y\in A\}$.

$\bar{A}$: closure of the set A.
$\bar{A}^{\Omega'}$: closure of the set A, relative to the induced topology on Ω'.
$B(x,\epsilon)$: open ball with center x and radius ϵ in a metric space.
$\mathrm{supp}(\phi)$: support of ϕ, closure of $\{\phi \neq 0\}$.
$C^c_{\mathbf{K}}(\Omega)$: set of $\mathbf{K}$-valued continuous maps with compact support.

14. Tutorial 14

$|b|$: total variation map of $b : \mathbf{R}^+ \to \mathbf{C}$.
$|b(t)|$: modulus of complex number $b(t)$.
$|b|(t)$: total variation of b evaluated at $t \in \mathbf{R}^+$.
$|f(t)|$: modulus of complex number $f(t)$.
$\mathcal{B}(\mathbf{R}^+)$, $\mathcal{B}(\mathbf{C})$: Borel σ-algebras on $\mathbf{R}^+$ and $\mathbf{C}$.
ds : Lebesgue measure on $(\mathbf{R}^+, \mathcal{B}(\mathbf{R}^+))$.
$|b|^+$: positive variation of b.
$|b|^-$: negative variation of b.
db : complex Stieltjes measure associated with b.
b^T : stopped map defined by $b^T(t) = b(t \wedge T)$.

$C^c_{\mathbf{C}}(\mathbf{R}^+)$: **C**-valued continuous maps on $\mathbf{R}^+$ with compact support.
$C^b_{\mathbf{C}}(\mathbf{R}^+)$: **C**-valued continuous maps on $\mathbf{R}^+$ which are bounded.
$b(t-)$: left-limit of b at t.
$\Delta b(t)$: jump of b at t, $\Delta b(t) = b(t) - b(t-)$.

15. Tutorial 15

$d|b|$: Stieltjes measure on $\mathbf{R}^+$ associated with total variation $|b|$.
$L^1_{\mathbf{C}}(b)$: **C**-valued, measurable maps f with $\int_{\mathbf{R}^+} |f|d|b| < +\infty$.
$L^{1,loc}_{\mathbf{C}}(b)$: measurable maps with $\int_0^t |f|d|b| < +\infty$ for all $t \in \mathbf{R}^+$.
$\int_0^t \dots$: partial Lebesgue integral on interval $[0, t]$.
$|db|$: total variation of complex Stieltjes measure db.
$t_n \downarrow\downarrow t$: $t < t_{n+1} \leq t_n$ for all $n \geq 1$, and $t = \inf_{n\geq 1} t_n$.
da : Stieltjes measure on $\mathbf{R}^+$ associated with a.
$f.a$: the map defined by $(f.a)(t) = \int_0^t f da$.
$d(f.a)$: Stieltjes measure on $\mathbf{R}^+$ associated with $f.a$.
a^T : stopped map defined by $a^T = a(t \wedge T)$.

$d(f.a)^T$: Stieltjes measure on $\mathbf{R}^+$ associated with $(f.a)^T$.
$|d(f.a)^T|$: total variation of measure $d(f.a)^T$.
$\Delta a(t)$: jump of a at t, $\Delta a(t) = a(t) - a(t-)$.
$d|b| << da$: $d|b|$ is absolutely continuous w.r. to da.

16. Tutorial 16

$\mathcal{B}(\Omega)$: Borel σ-algebra on Ω.
$L^1_{\mathbf{R}}(\Omega, \mathcal{B}(\Omega), \mu)$: real valued Borel measurable f's with $\int |f| d\mu < +\infty$.
$\mathcal{T}_{|A}$: induced topology on A, $\mathcal{T}_{|A} = \{A \cap V : V \in \mathcal{T}\}$.
$\mathcal{T}_{\mathbf{R}}$: usual topology on $\mathbf{R}$.
$|\mu|$: total variation of complex measure μ.
$M\mu$: maximal function of complex measure μ.
$B(x, \epsilon)$: open ball with center x and radius ϵ.
$\mathbf{N}_p$: the set $\{1, \ldots, p\}$.
$\|\mu\|$: total mass of total variation, $\|\mu\| = |\mu|(\mathbf{R}^n)$.
Mf : maximal function of f.

$dx(B(x,\epsilon))$: Lebesgue measure of open ball $B(x,\epsilon)$ in $\mathbf{R}^n$.

17. Tutorial 17

$\mathbf{K}$: the field $\mathbf{R}$ or $\mathbf{C}$.
$\mathcal{M}_n(\mathbf{K})$: set of $n \times n$ matrices with $\mathbf{K}$-valued entries.
$e_1,\ldots,e_n$: canonical basis of $\mathbf{K}^n$.
μ^X, $X(\mu)$: law, distribution of X under μ, image measure of μ by X.
$X^{-1}(B)$, $\{X \in B\}$: inverse image of B by X.
$Y \circ X$: composition of X and Y, $(Y \circ X)(\omega) = Y(X(\omega))$.
τ_a : translation mapping of vector a in $\mathbf{R}^n$.
$\uplus$: union of pairwise disjoint sets.
$\mathcal{B}(\mathbf{R}^n)$: Borel σ-algebra on $\mathbf{R}^n$.
$\sigma(\mathcal{C})$: σ-algebra on $\mathbf{R}^n$ generated by $\mathcal{C}$.
dx : Lebesgue measure on $\mathbf{R}^n$.
$\det \Sigma$: determinant of matrix Σ.
$\dim V$: dimension of liear subspace V of $\mathbf{R}^n$.

18. Tutorial 18

$\mathbf{K}$: the field $\mathbf{R}$ or $\mathbf{C}$.
$N, \|\cdot\|$: norm on a $\mathbf{K}$-vector space.
E, F : $\mathbf{K}$-normed spaces.
$\mathcal{L}_K(E, F)$: set of continuous linear maps $l : E \to F$.
$d\phi(a)$: differential of ϕ at a.
$d\phi$: differential mapping of ϕ.
$\frac{\partial\phi}{\partial x_i}(a)$: i-th partial derivative of ϕ at a.
$l_{|U}$: restriction of l to U.
$J(\phi)(a)$: jacobian of ϕ at a, determinant of $d\phi(a)$.
$\mathcal{B}(\mathbf{R}^n)$: Borel σ-algebra on $\mathbf{R}^n$.
$dx_{|\Omega}$: Lebesgue measure on $\Omega \in \mathcal{B}(\mathbf{R}^n)$, restriction of dx to $\mathcal{B}(\Omega)$.
$B(a, \epsilon)$: open ball with center a and radius ϵ.
$\phi(dx_{|\Omega})$: image measure of $dx_{|\Omega}$ by ϕ, $\phi(dx_{|\Omega})(B) = dx_{|\Omega}(\phi^{-1}(B))$.
$\int |J(\psi)| dx_{|\Omega'}$: measure on Ω' with density $|J(\psi)|$ w.r. to $dx_{|\Omega'}$.

19. Tutorial 19

$C^1(\mathbf{R},\mathbf{R})$: real, continuously differentiable maps on $\mathbf{R}$.
$\mu_1 \star \ldots \star \mu_p$: the convolution of $\mu_1, \ldots, \mu_p$.
$\mu \star \nu$: the convolution of μ and ν.
$\mu \otimes \nu$: the product measure of μ and ν.
$B - x$: the set $\{y \in \mathbf{R}^n : y + x \in B\}$.
δ_a : dirac probability measure on $\mathbf{R}^n$, centered in $a \in \mathbf{R}^n$.
τ_a : translation mapping on $\mathbf{R}^n$, $\tau_a(x) = a + x$.
$\mathcal{B}(\mathbf{R}^n) \otimes \mathcal{B}(\mathbf{R}^n)$: product of Borel σ-algebras on $\mathbf{R}^n \times \mathbf{R}^n$.
$\mathcal{F}\mu$: Fourier transform of complex measure μ.
$C^b_{\mathbf{R}}(\Omega)$: set of real functions on Ω, which are continuous and bounded.
$\mu_k \to \mu$, narrowly : for all $f \in C^b_{\mathbf{R}}(\Omega)$, $\int f d\mu_k \to \int f d\mu$.
ϕ_X : characteristic function of $\mathbf{R}^n$-valued random variable X.
$|\alpha|$: for $\alpha \in \mathbf{N}^n$, $|\alpha| = \alpha_1 + \ldots + \alpha_n$.
x^α : for $\alpha \in \mathbf{N}^n$ and $x \in \mathbf{R}^n$, $x^\alpha = x_1^{\alpha_1} \ldots x_n^{\alpha_n}$.
$\partial^\alpha f$: the $|\alpha|$-th order partial derivative of f, $\partial^\alpha f = \frac{\partial^{|\alpha|} f}{\partial x_1^{\alpha_1} \ldots \partial x_n^{\alpha_n}}$.
$x^\alpha \mu$: $x^\alpha \mu = \int x^\alpha d\mu$, measure with density x^α w.r. to μ.

20. Tutorial 20

$\mathcal{M}_n(\mathbf{R})$: set of $n \times n$ matrices with real entries.
M^t : transposed matrix of M.
M^{-1} : inverse matrix of non-singular matrix M.
$\langle u, Mu \rangle$: inner-product in $\mathbf{R}^n$ of u and Mu.
Σ : a symmetric and non-negative $n \times n$ real matrix.
$\phi(\mu)$: image measure of μ by ϕ, $\phi(\mu)(B) = \mu(\phi^{-1}(B))$.
$\mathcal{F}P(u)$: Fourier transform of probability P, evaluated at u.
$N_n(m, \Sigma)$: Gaussian measure on $\mathbf{R}^n$ with mean m and covariance Σ.
$N_1(0, 1)$: reduced Gaussian measure on $\mathbf{R}$.
x^α : for $\alpha \in \mathbf{N}^n$ and $x \in \mathbf{R}^n$, $x^\alpha = x_1^{\alpha_1} \ldots x_n^{\alpha_n}$.
$cov(X, Y)$: covariance between square-integrable variables X and Y.
$var(X)$: variance of square-integrable random variable X.
δ_0, δ_1 : dirac probability measures on $\mathbf{R}$, centered in 0 and 1.
$\det \Sigma$: determinant of matrix Σ.
dx : Lebesgue measure on $\mathbf{R}^n$.